THE ELASTIC EQUILIBRIUM OF AN INFINITE, TRANSVERSELY ISOTROPIC BODY, WEAKENED BY AN INTERNAL FLAT CIRCULAR CUT

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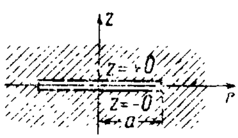
The paper studies the deformation, symmetrical about the plane $z = 0$, of an infinite transversely isotropic body containing an internal flat circular slot. The same problem for an isotropic medium has been investigated by a different method in [1].

1. Suppose that an infinitely thin flat circular slot $z = 0$, $r \leqslant a$ is located with its centre at the origin of co-ordinates in an infinite transversely isotropic space (see figure).



Suppose that an external load is applied to the surface of the slot. The boundary conditions are then

$$\sigma_z|_{z=+0} = \sigma_z|_{z=-0} = \sigma(r, \varphi) \qquad (r < a) \tag{1.1}$$

$$(\tau_{rz} + \tau_{\varphi z})_{z=+0} = -(\tau_{rz} + i\tau_{\varphi z})_{z=-0} = \tau_1(r, \varphi) (r < a)$$

$$(\tau_{rz} - i\tau_{\varphi z})_{z=+0} = -(\tau_{rz} - i\tau_{\varphi z})_{z=-0} = \tau_2(r, \varphi) \qquad (r < a)$$

Symmetry at the section $z = 0$ leads to the further conditions

$$U_3|_{z=0} = 0 \quad (r > a), \qquad (\tau_{rz} \pm i\tau_{\varphi z})_{z=0} = 0 \qquad (r > a) \tag{1.2}$$

It will be shown later that it is expedient to introduce complex stress components; this is associated with the proposed method of solution.

If we consider that the plane $z = 0$ divides the space into two half-spaces, we can reduce the problem to two boundary-value problems.

(A). For the half-space $z \geqslant 0$

$$\sigma_z|_{z=0} = \sigma(r, \varphi) \quad (r < a), \qquad U_3|_{z=0} = 0 \quad (r > a)$$

$$(\tau_{rz} + i\tau_{\varphi z})_{z=0} = \tau_1(r, \varphi) = \begin{cases} 0 & (r > a) \\ \tau_1(r, \varphi) & (r < a) \end{cases}$$

$$(\tau_{rz} - i\tau_{\varphi z})_{z=0} = \tau_2(r, \varphi) = \begin{cases} 0 & (r > a) \\ \tau_2(r, \varphi) & (r < a) \end{cases} \tag{1.3}$$

(B). For the half-space $z \leqslant 0$

$$
\begin{gathered}
\sigma_z|_{z=0} = \sigma(r, \varphi) \quad (r < a) \qquad U_3|_{z=0} = 0 \quad (r > a) \\
(\tau_{rz} + i\tau_{\varphi z})_{z=0} = -\tau_1(r, \varphi) = \begin{cases} 0 & (r > a) \\ -\tau_1(r, \varphi) & (r < a) \end{cases} \\
(\tau_{rz} - i\tau_{\varphi z})_{z=0} = -\tau_2(r, \varphi) = \begin{cases} 0 & (r > a) \\ -\tau_2(r, \varphi) & (r < a) \end{cases}
\end{gathered} \tag{1.4}
$$

Assuming that the boundary functions can be expanded in a Fourier series in φ and allow a Hankel integral transform in r, the boundary-values problems (A) and (B) lead to well-known dual integral equations which have an exact solution.

The functions of the problem are obtained by the method of total separation of variables in the system of equations of the theory of elasticity for a transversely isotropic medium.

2. In [2] the author has used the proposed method to derive certain general expressions for the elastic displacements of a transversely isotropic heterogeneous medium. The same method was used in [3] to obtain a class of solutions to the static equations of the theory of elasticity for a transversely isotropic homogeneous medium. We start from the familiar generalized Hooke's law [4] for a homogeneous transversely isotropic medium

$$
\begin{aligned}
\sigma_x &= a_{11}\frac{\partial U}{\partial x} + a_{12}\frac{\partial V}{\partial y} + a_{13}\frac{\partial W}{\partial z}, \qquad & \tau_{xz} &= a_{55}\left(\frac{\partial W}{\partial x} + \frac{\partial U}{\partial z}\right) \\
\sigma_y &= a_{12}\frac{\partial U}{\partial x} + a_{11}\frac{\partial V}{\partial y} + a_{13}\frac{\partial W}{\partial z}, \qquad & \tau_{yz} &= a_{55}\left(\frac{\partial W}{\partial y} + \frac{\partial V}{\partial z}\right) \\
\sigma_z &= a_{13}\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) + c_{33}\frac{\partial W}{\partial z}, \qquad & \tau_{xy} &= a_{66}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)
\end{aligned} \tag{2.1}
$$

Here σ_s are the normal stresses, τ_{s_2} are the shear stresses, U, V and W are the components of displacement along the co-ordinate axes and $a_{11} - a_{12} = 2a_{66}$.

By substituting the values of stresses into the Cauchy system of equilibrium equations, we obtain the elasticity equations in displacements

$$
\begin{gathered}
a_{66}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) + a_{55}\frac{\partial^2 U}{\partial z^2} + \frac{\partial}{\partial x}\left[\frac{a_{11}+a_{12}}{2}\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) + (a_{13}+a_{55})\frac{\partial W}{\partial z}\right] = 0 \\
a_{66}\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) + a_{55}\frac{\partial^2 V}{\partial z^2} + \frac{\partial}{\partial y}\left[\frac{a_{11}+a_{12}}{2}\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) + (a_{13}+a_{55})\frac{\partial W}{\partial z}\right] = 0 \\
a_{55}\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) + a_{33}\frac{\partial^2 W}{\partial z^2} + (a_{13}+a_{55})\frac{\partial}{\partial z}\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) = 0
\end{gathered} \tag{2.2}
$$

Using cylindrical coordinates and replacing the displacements with new functions defined by the formulas

$$
U = \frac{1}{2}(e^{i\varphi}U_1 + e^{-i\varphi}U_2), \qquad V = -\frac{i}{2}(e^{i\varphi}U_1 - e^{-i\varphi}U_2), \qquad W = U_3 \tag{2.3}
$$

we obtain a system of equations in U_l

$$
\begin{gathered}
a_{66}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{2i}{r^2}\frac{\partial}{\partial \varphi} - \frac{1}{r^2}\right)U_1 + a_{55}\frac{\partial^2 U_1}{\partial z^2} + \left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \varphi}\right)\left\{\frac{a_{11}+a_{12}}{4}\times\right. \\
\left.\times\left[\left(\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial \varphi} + \frac{1}{r}\right)U_1 + \left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \varphi} + \frac{1}{r}\right)U_2\right] + (a_{13}+a_{55})\frac{\partial U_3}{\partial z}\right\} = 0
\end{gathered} \tag{2.4}
$$

$$a_{66}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}-\frac{2i}{r^2}\frac{\partial}{\partial\varphi}-\frac{1}{r^2}\right)U_2+a_{55}\frac{\partial^2 U_2}{\partial z^2}+\left(\frac{\partial}{\partial r}-\frac{i}{r}\frac{\partial}{\partial\varphi}\right)\times$$

$$+\left\{\frac{a_{11}+a_{12}}{4}\left[\left(\frac{\partial}{\partial r}-\frac{i}{r}\frac{\partial}{\partial\varphi}+\frac{1}{r}\right)U_1+\left(\frac{\partial}{\partial r}+\frac{i}{r}\frac{\partial}{\partial\varphi}+\frac{1}{r}\right)U_2\right]+(a_{13}+a_{55})\frac{\partial U_3}{\partial z}\right\}=0$$

$$a_{55}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right)U_3+a_{33}\frac{\partial^2 U_3}{\partial z^2}+$$

$$+\frac{a_{13}+a_{55}}{2}\frac{\partial}{\partial z}\left[\left(\frac{\partial}{\partial r}-\frac{i}{r}\frac{\partial}{\partial\varphi}+\frac{1}{r}\right)U_1+\left(\frac{\partial}{\partial r}+\frac{i}{r}\frac{\partial}{\partial\varphi}+\frac{1}{r}\right)U_2\right]=0$$

The following simple relation holds between the displacement components along the cylindrical coordinate axes and the functions U_l

$$U_1=U_r-iU_\varphi,\qquad U_2=U_r+iU_\varphi,\qquad U_3=W \tag{2.5}$$

The formulas which relate the stress components σ_z and $\tau_{rz}\pm i\tau_{\varphi z}$ to the function U_l are

$$\sigma_z=a_{33}\frac{\partial U_3}{\partial z}+\frac{a_{13}}{2}\left[\left(\frac{\partial}{\partial r}-\frac{i}{r}\frac{\partial}{\partial\varphi}+\frac{1}{r}\right)U_1+\left(\frac{\partial}{\partial r}+\frac{i}{r}\frac{\partial}{\partial\varphi}+\frac{1}{r}\right)U_2\right] \tag{2.6}$$

$$\tau_{rz}+i\tau_{\varphi z}=a_{55}\left[\frac{\partial U_1}{\partial z}+\left(\frac{\partial U_3}{\partial r}+\frac{i}{r}\frac{\partial U_3}{\partial\varphi}\right)\right],\qquad \tau_{rz}-i\tau_{\varphi z}=a_{55}\left[\frac{\partial U_2}{\partial z}+\left(\frac{\partial U_3}{\partial r}-\frac{i}{r}\frac{\partial U_3}{\partial\varphi}\right)\right]$$

We seek a particular solution to (2.4) in the form

$$U_1=J_{k+1}(\alpha r)X_1(z)e^{ik\varphi},\qquad U_2=J_{k-1}(\alpha r)X_2(z)e^{ik\varphi},\quad U_3=J_k(\alpha r)X_3(z)e^{ik\varphi} \tag{2.7}$$

Substitution of (2.7) into (2.4) leads immediately to a system of ordinary differential equations

$$\begin{gathered}a_{55}x_1''-\left(a_{11}-\frac{a_{11}+a_{12}}{4}\right)\alpha^2x_1+\frac{a_{11}+a_{12}}{4}\alpha^2x_2-(a_{13}+a_{55})\alpha x_3'=0\\ a_{55}x_2''-\left(a_{11}-\frac{a_{11}+a_{12}}{4}\right)\alpha^2x_2+\frac{a_{11}+a_{12}}{4}\alpha^2x_1+(a_{13}+a_{55})\alpha x_3'=0\\ a_{33}x_3''-a_{55}\alpha^2x_3+\frac{a_{13}+a_{55}}{2}\alpha(x_1'-x_2')=0\end{gathered} \tag{2.8}$$

We express the characteristic numbers in the form

$$\pm\alpha\lambda_1,\qquad \pm\alpha\lambda_3\qquad \pm\alpha\lambda_5$$

Here λ_l are the positive roots of the equation

$$(a_{55}\lambda^2-a_{66})\,[a_{33}a_{55}\lambda^4-(a_{11}a_{33}+2a_{13}a_{55}-a_{13}^2)\lambda^2+a_{11}a_{55}]=0 \tag{2.9}$$

$$\lambda_1^2=a_{66}\,/\,a_{55}$$

$$\begin{gathered}\lambda_3^2=\frac{1}{2a_{55}a_{33}}\left\{a_{11}a_{33}+2a_{13}a_{55}-a_{13}^2+\sqrt{(a_{11}a_{33}^2+2a_{13}a_{55}-a_{13}^2)^2-4a_{11}a_{33}a_{55}^2}\right\}\\ \lambda_5^2=\frac{1}{2a_{55}a_{33}}\left\{a_{11}a_{33}+2a_{13}a_{55}-a_{13}^2-\sqrt{(a_{11}a_{33}^2+2a_{13}a_{55}-a_{13}^2)^2-4a_{11}a_{33}a_{55}^2}\right\}\end{gathered} \tag{2.10}$$

For these characteristic numbers the most general solution of (2.4) can be written as

$$U_1=\sum_{k=-\infty}^{+\infty}e^{ik\varphi}\int_0^\infty J_{k+1}(\alpha r)\sum_{l=1,3,5}(C_le^{\alpha\lambda_lz}+C_{l+1}e^{-\alpha\lambda_lz})\,d\alpha$$

$$U_2=\sum_{k=-\infty}^{+\infty}e^{ik\varphi}\int_0^\infty J_{k-1}(\alpha r)\left\{C_1e^{\alpha\lambda_1z}+C_2e^{-\alpha\lambda_1z}-\sum_{l=3,5}(C_le^{\alpha\lambda_lz}+C_{l+1}e^{-\alpha\lambda_lz})\right\}d\alpha \tag{2.11}$$

$$U_3=\sum_{k=-\infty}^{+\infty} e^{ik\varphi}\int_0^\infty J_k(\alpha r)\sum_{l=3,5}\frac{a_{55}\lambda_l^2-a_{11}}{\lambda_l(a_{13}+a_{55})}(C_l e^{\alpha\lambda_l z}-C_{l+1}e^{-\alpha\lambda_l z})\,d\alpha$$

Substituting the values of the U_l into (2.6) yields

$$\sigma_z=\sum_{k=-\infty}^{+\infty} e^{ik\varphi}\int_0^\infty \alpha J_k(\alpha r)\Big\{\sum_{l=3,5}\Big[\frac{a_{33}(a_{55}\lambda_l^2-a_{11})}{(a_{13}+a_{55})}+C_{13}\Big](C_l e^{\alpha\lambda_l z}+C_{l+1}e^{-\alpha\lambda_l z})\Big\}\,d\alpha$$

$$\tau_{rz}+i\tau_{\varphi z}=a_{55}\sum_{k=-\infty}^{+\infty} e^{ik\varphi}\int_0^\infty \alpha J_{k+1}(\alpha r)\Big\{\alpha_1(C_1e^{\alpha\lambda_1 z}-C_2e^{-\alpha\lambda_1 z})+$$

$$+\sum_{l=3,5}\frac{a_{13}\lambda_l^2+a_{11}}{\lambda_l(a_{13}+a_{55})}(C_l e^{\alpha\lambda_l z}-C_{l+1}e^{-\alpha\lambda_l z})\Big\}\,d\alpha \qquad (2.12)$$

$$\tau_{rz}-i\tau_{\varphi z}=a_{55}\sum_{k=-\infty}^{+\infty} e^{ik\varphi}\int_0^\infty \alpha J_{k-1}(\alpha r)\Big\{\lambda_1(C_1e^{\alpha\lambda_1 z}-C_2e^{-\alpha\lambda_1 z})-$$

$$-\sum_{l=3,5}\frac{a_{13}\lambda_l^2+a_{11}}{\lambda_l(a_{13}+a_{55})}(C_l e^{\alpha\lambda_l z}-C_{l+1}e^{-\alpha\lambda_l z})\Big\}\,d\alpha$$

3. From now on we shall assume that the boundary functions can be expanded in a Fourier series in φ and allow a Hankel integral transform in r. With these assumptions we can solve the following more general mixed boundary-value problem for a transversely isotropic elastic half-space $x_3\geqslant 0$ with the boundary conditions

$$U_3|_{z=0}=U(r,\varphi)=\sum_{k=-\infty}^{+\infty}U_k(r)\,e^{ik\varphi}\qquad (r>a)$$

$$\sigma_z|_{z=0}=\sigma(r,\varphi)=\sum_{k=-\infty}^{+\infty}\sigma_k(r)\,e^{ik\varphi}\qquad (r<a) \qquad (3.1)$$

$$(\tau_{rz}+i\tau_{\varphi z})_{z=0}=\tau_1(r,\varphi)=\sum_{k=-\infty}^{+\infty}e^{ik\varphi}\int_0^\infty d\tau_{1k}{}^\circ(\alpha)\,J_{k+1}(\alpha r)\,d\alpha$$

$$(\tau_{rz}-i\tau_{\varphi z})_{z=0}=\tau_2(r,\varphi)=\sum_{k=-\infty}^{+\infty}e^{ik\varphi}\int_0^\infty d\tau_{2k}{}^\circ(\alpha)\,J_{k-1}(\alpha r)\,d\alpha \qquad (3.2)$$

To solve this problem we shall use Formulas (2.11) and (2.12), putting $C_l=0$ ($l=1$, 3, 5). Having satisfied the boundary conditions (3.1) and (3.2) we obtain a system of equations for determining C_{l+1}, which we write in the expanded form

$$\int_0^\infty J_k(\alpha r)\Big\{\frac{a_{55}\lambda_3^2-a_{11}}{\lambda_3(a_{13}+a_{55})}C_4+\frac{a_{55}\lambda_5^2-a_{11}}{\lambda_5(a_{13}+a_{55})}C_6\Big\}\,d\alpha=-U_k(r)\qquad (r>a)$$

$$\int_0^\infty \alpha J_k(\alpha r)\Big\{\Big[\frac{a_{33}(a_{55}\lambda_3^2-a_{11})}{(a_{13}+a_{55})}+a_{13}\Big]C_4+\Big[\frac{a_{33}(a_{55}\lambda_5^2-a_{11})}{a_{13}+a_{55}}+a_{13}\Big]C_6\Big\}\,d\alpha=\sigma_k(r) \qquad (3.3)$$

$$(r<a)$$

$$-a_{55}\Big\{\lambda_1C_2+\frac{a_{13}\lambda_3^2+a_{11}}{\lambda_3(a_{13}+a_{55})}C_4+\frac{a_{13}\lambda_5^2+a_{11}}{\lambda_5(a_{13}+a_{55})}C_6\Big\}=\mathring{\tau}_{1k}(\alpha) \qquad (3.4)$$

$$a_{55}\left\{\lambda_1 C_2 - \frac{a_{13}\lambda_3^2 + a_{11}}{\lambda_3 (a_{13} + a_{55})} C_4 - \frac{a_{13}\lambda_5^2 + a_{11}}{\lambda_5 (a_{13} + a_{55})} C_6\right\} = \tau_{2k}^{\circ} (\alpha)$$

For brevity we introduce the notations

$$m_1 = \frac{a_{55}\lambda_3^2 - a_{11}}{\lambda_3 (a_{13} + a_{55})}, \qquad m_2 = \frac{a_{55}\lambda_5^2 - a_{11}}{\lambda_5 (a_{13} + a_{55})}$$

$$m_3 = \frac{a_{33} (a_{55}\lambda_3^2 - a_{11})}{(a_{13} + a_{55})} + a_{13}, \qquad m_4 = \frac{a_{33} (a_{55}\lambda_5^2 - a_{11})}{(a_{13} + a_{55})} + a_{13} \tag{3.5}$$

$$m_5 = \frac{(a_{13}\lambda_3^2 + a_{11}) a_{55}}{\lambda_3 (a_{13} + a_{55})}, \qquad m_6 = \frac{(a_{13}\lambda_5^2 + a_{11}) a_{55}}{\lambda_5 (a_{13} + a_{55})}$$

We have from (3.4) that

$$C_2 = -\frac{\tau_{1k}^{\circ} (\alpha) - \tau_{2k}^{\circ} (\alpha)}{2C_{55}} \tag{3.6}$$

$$C_4 = -\frac{m_6}{m_5} C_6 - \frac{\tau_{1k}^{\circ} (\alpha) + \tau_{2k}^{\circ} (\alpha)}{2m_5} \tag{3.7}$$

By means of (3.7) we eliminate C_4 from (3.3) and thus obtain the dual integral equations

$$\int_0^\infty J_k (\alpha r) C_6 (\alpha) \, d\alpha = f_1 (r) \quad (r > a), \qquad \int_0^\infty \alpha J_k (\alpha r) C_6 (\alpha) \, d\alpha = f_2 (r) \quad (r < a) \tag{3.8}$$

$$f_1 (r) = \frac{1}{m_1 m_6 - m_2 m_5} \left\{m_5 U_k (r) - \frac{m_1}{2} \int_0^\infty [\tau_{1k}^{\circ} (\alpha) + \tau_{2k}^{\circ} (\alpha)] J_k (\alpha r) \, d\alpha\right\}$$

$$f_2 (r) = \frac{1}{m_4 m_5 - m_3 m_6} \left\{m_4 \sigma_k + \frac{m_3}{2} \int_0^\infty [\tau_{1k}^{\circ} (\alpha) + \tau_{2k}^{\circ} (\alpha)] J_k (\alpha r) \, d\alpha\right\}$$

Assuming that $f_1 (r) \equiv 0$, we obtain its Hankel transform for $r < a$:

$$\alpha g (\alpha) = \int_a^\infty f_1 (r) J_k (\alpha r) \, dr$$

Instead of $C_6 (\alpha)$ we introduce a new unknown function $f_k (\alpha)$ by means of the formula

$$C_6 (\alpha) = f_k^{\circ} (\alpha) + \alpha g (\alpha) \tag{3.9}$$

and thus obtain the familiar dual integral equations

$$\int_0^\infty f_k^{\circ} (\alpha) J_k (\alpha r) \, d\alpha = 0 \quad (r > a), \qquad \int_0^\infty \alpha J_k (\alpha r) f_k (\alpha) \, d\alpha = F_k (r) \quad (r < a) \tag{3.10}$$

$$F_k (r) = \int_0^\infty \alpha^2 g (\alpha) J_k (\alpha r) \, d\alpha + f_2 (r) \tag{3.11}$$

We quote the exact solution of the dual integral equations from the monograph [5]:

$$f_k^\circ(\alpha) = \frac{\sqrt{2\alpha}}{\Gamma(1/2)} \int_0^a t^{3/2} J_{k+1/2}(t\alpha)\, dt \int_0^a \frac{F_k(t\lambda)\lambda^{k+1}}{\sqrt{a^2-\lambda^2}}\, d\lambda \tag{3.12}$$

Expressing $C_4(\alpha)$ and $C_6(\alpha)$ in terms of $f_k{}^\circ(\alpha)$ and $g(\alpha)$ we obtain a solution to the problems in general form.

Returning to the problems (A) and (B), we easily see that they represent a particular case of the more general boundary-value problem solved in section 3. When solving the problem (A) we assume $C_l = 0$ ($l = 1, 3, 5$) in formulas (2.11) and (2.12), while when solving the problem (B) we have $C_{l+1} = 0$. Since in problems (A) and (B) $U_3|_{z=0} = 0$, the calculations are somewhat simplified.

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